# *«Binary Hypothesis testing»*

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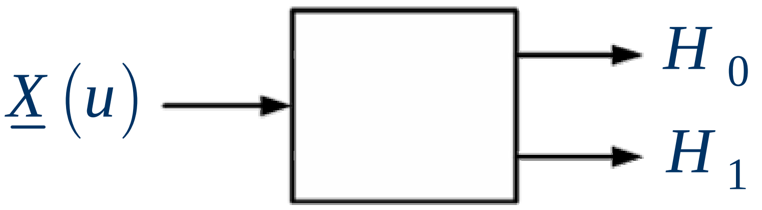
# Binary Hypothesis-Testing

Suppose we observe some data forming a random vector $\underline{X}(u)$ and we would like to decide between two alternatives which we call hypotheses.

$$H_1 : \underline{X}(u) = \underline{s}_1 + \underline{N}(u)$$

$$H_0 : \underline{X}(u) = \underline{s}_0 + \underline{N}(u)$$

We would like to come up with a decision rule which takes $\underline{X}(u)$ as input and produces a decision at the output.



# Binary Hypothesis-Testing

- Under certain conditions a perfect decision can be made (singular detection). For example

i. There is no noise present, i.e., $\underline{N}(u) = \underline{0}$

ii. Noise is in a different direction than the signals, e.g.,

$$\underline{s}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ \underline{s}_0 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \ \underline{N}(u) = \begin{bmatrix} N_1(u) \\ 0 \end{bmatrix}$$

iii. Noise values are bounded, e.g.,

$$\Pr\left\{ \left\| \underline{N}(u) \right\| \geq a \right\} = 0 \quad \text{and} \quad \left\| \underline{s}_1 - \underline{s}_0 \right\| > 2a$$

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- Assume that noise is white, i.e., $\underline{K}_N = \sigma^2 \underline{I}$. A reasonable detection rule is the minimum distance rule:

$$\left\| \underline{X}(u) - \underline{s}_1 \right\|^2 \underset{H_1}{<} \left\| \underline{X}(u) - \underline{s}_0 \right\|^2$$

- Expanding $\left\| \underline{X}(u) - \underline{s}_i \right\|^2$ as

$$\begin{aligned} \left\| \underline{X} - \underline{s}_i \right\|^2 &= \left( \underline{X} - \underline{s}_i \right)^{*T} \left( \underline{X} - \underline{s}_i \right) \\ &= \underline{X}^{*T} \underline{X} - \underline{s}_i^{*T} \underline{X} - \underline{X}^{*T} \underline{s}_i + \underline{s}_i^{*T} \underline{s}_i \\ &= \left| \underline{X} \right|^2 + \left| \underline{s}_i \right|^2 - 2 \operatorname{Re}\left\{ \underline{X}^{*T} \underline{s}_i \right\} \end{aligned}$$

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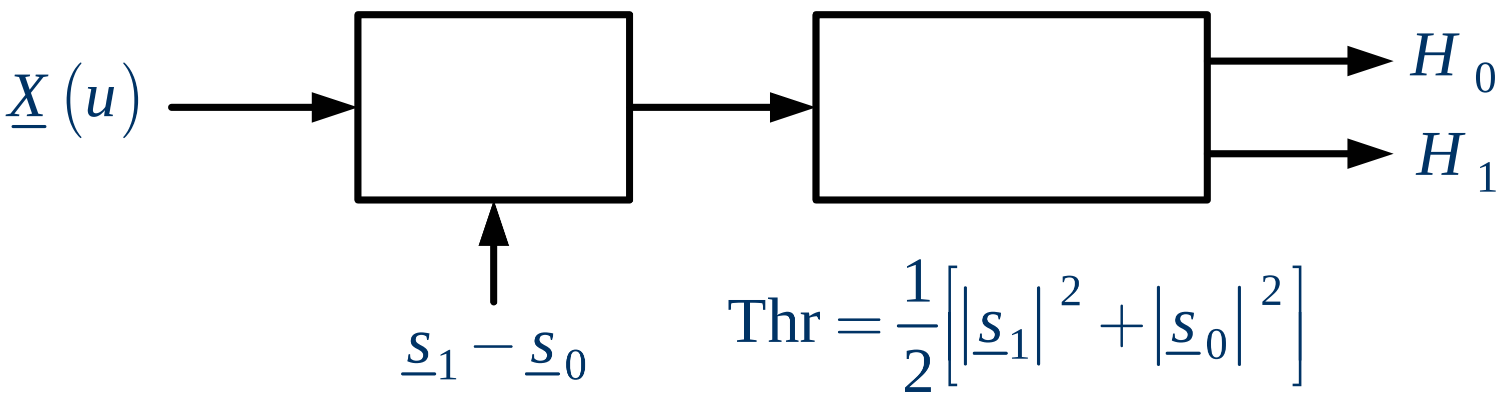
- The decision rule reduces to

$$\mathrm{Re}\left\{\underline{X}^{*T}\left(\underline{s}_1-\underline{s}_0\right)\right\}\ \underset{H_1}{>}\ \frac{1}{2}\left[\left|\underline{s}_1\right|^2+\left|\underline{s}_0\right|^2\right]$$




$$\mathrm{Thr}=\frac{1}{2}\left[\left|\underline{s}_1\right|^2+\left|\underline{s}_0\right|^2\right]$$